LEVEL
High schools after students have learned number systems conversion.

OBJECTIVES
To explore the four arithmetic operations with the four number systems in discussion: binary, octal, decimal and hexadecimal.

OVERVIEW
Arithmetic operations such as addition and multiplication are steps forward into understanding numbers of base-10; in this exploration we look into doing arithmetic not just for base-10 numbers but also for binary, octal and hexadecimal numbers systems. We also explore arithmetical calculations using numbers of different base.

EXPLORATORY ACTIVITIES
[Note]
(a) Unless otherwise specified, we choose MthIO mode in the SETUP menu by tapping

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SHIFT MODE 1 (MthIO)
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(b) The system each number represented will be described with a subscript of 2, 8, and 16 for binary, octal and hexadecimal respectively; except for number in base-10, which will be written just as it is.

Addition and Subtraction
We begin by reviewing the method we usually apply in adding two decimal numbers. We place digits within these two numbers into their respective ‘value column’, then add digits which are on the same ‘value column’. A unit is carry forward to the next column on left when the sum of these digits is > 10, while the remainder of this sum minus 10 is place in the original column. And the process goes on.

As an example, consider the addition 176 + 87. The process of adding up these two numbers can best be described by the following process-flow diagrams.
In a similar fashion, addition of two or even more binary numbers works in such way too; only carrying forward occurs when the sum is = 2. Adding 101₂ with 11₂ is as follow.

\[
\begin{array}{c}
101_2 + 11_2 \\
\hline
110_2
\end{array}
\]

Therefore, 101₂ + 11₂ is equivalent to 1000₂. We can do this addition fairly easy in the 991-ES.

[Operations]
- Enter Base-N mode
- Switch to the Bin, or binary system
- Enter 101₂ into the calculator
- Enter 11₂ for the addition
- Now evaluate the addition

The Base-N mode verifies the addition of 101₂ + 11₂ as 1000₂.

Performing addition with octal and hexadecimal systems are very much the same as with doing it in decimal or binary. For example addition of 357₈ + 701₃₈ is as follow.

\[
\begin{array}{c}
7063 + 357 \\
\hline
7063 + 357 \\
\hline
7063 + 357 \\
\hline
7063 + 357
\end{array}
\]

Carrying forward occurs when the sum of digits is > 8, and with such we have found that the addition of 357₈ + 701₃₈ is 7442₂.
Subtraction between two numbers of any systems is very similar to addition; numbers are placed accordingly to the ‘value column’ and subtraction is performed between the digits placed on the same column. ‘Borrowing’ is needed when the digit above is smaller than the one below. For example, to subtract 2B5₁₆ from A6₁₆,

\[
\begin{array}{c}
A \ 6 \ 1 \\
- \ 2 \ B \ 5 \\
\hline
\end{array}
\quad \Rightarrow \quad \begin{array}{c}
A \ _{-1} \ 6 \ _{-1}^{16} \\
- \ 2 \ B \ 5 \\
\hline
A \ _{-2} \ 6 \ _{-1}^{16} \\
\end{array}
\quad \Rightarrow \quad \begin{array}{c}
A \ 6 \ 1 \\
- \ 2 \ B \ 5 \\
\hline
A \ 6 \ 1 \\
\end{array}
\]

As you can see the subtraction for hexadecimal is very similar to the decimal. In this calculation we have shown that A6₁₆ – 2B5₁₆ = 7AC₁₆ and we can check the result on the 991-ES.

**[Operations]**
- Enter Base-N mode
- Switch to the Hex system
- Enter A6₁₆ – 2B5₁₆
- Now evaluate the subtraction

**Using the Calculator**
The use of calculator should be for speeding up arithmetic calculations and not for replacing understanding of the concepts discussed. Special attentions have to be given while using 991-ES as we look at examples of using the calculator to perform variation of addition, subtraction as we also explore doing multiplication and division.

**Example 1:** Find 137₅₈ – 517₈ + 336₈.

**[Operations]**
- Enter Base-N then Oct mode
- Enter 137₈
- Then enter ‘– 517’
- Follow by +336₈
- Now evaluate

The result of 137₅₈ – 517₈ + 336₈ is 1214₈.

In the next example, we want to calculate the difference of 101₁₂ and 110010₁₂, but we will put the smaller number at front.
Example 2: Find $1011_2 - 1100101_2$.

[Operations]
- Enter Base-N mode
- Switch to the Bin mode
- Enter $1011_2$
- Followed by $1100101_2$
- Now evaluate

The result displayed is definitely not the correct solution. The reason is because the calculation actually produces a negative value and this causes an overflow to the calculator. Thus special care has to be given while performing subtraction in the Base-N mode as it is not designed to do negative binary, octal or hexadecimal values.

Example 3: Find the product of $7712_8 \times 356_8$.

[Operations]
- Enter Base-N mode
- Switch to the Oct mode
- Enter $7712_8$
- Multiply by $356_8$
- Now evaluate

The product is $3526714_8$. And suppose you are interested in finding what this number translates to in decimal, just tap $\chi^4$ to switch to Dec mode and you will find out.

Finally, we could actually perform division in 991-ES but again care has to be given when doing the division.

Example 4: Find (i) $3A8_{16} \div C2_{16}$.

[Operations]
- Enter Base-N then Hex mode
- Enter $3A8_{16} \div C2_{16}$
- Now evaluate

The output of $4_{16}$ could not be right, as $3A8_{16} \div C2_{16} = 936 \div 194 \approx 4.82$. Therefore you should ensure the result of the division must be without remainder such as the division of $B4_{16} \div 3C_{16}$.
Cross Base Arithmetic Operation
Operation involving different bases is possible on 991-ES. For example, consider the following.

\[(719 + 7C_{16}) \div 1101_2 = ?\]

Before we begin doing this in the calculator, we should decide in what base should the result be displayed? For this example let’s choose to display the result in base-8.

[Operations]
- Enter Base-N mode
- Display in Oct mode
- Enter base defining mode
- Define 719 as base-10 for addition
- Define 7C_{16} as base-16
- Now evaluate \((719 + 7C_{16})\)
- Use Answer memory for division
- Now evaluate Ans \div 1101_2

In Oct mode the result is 320_8. In decimal the result is 208.

EXERCISES
These exercises should enhance your understanding on the arithmetic operations with number systems. Calculate each one manually before verifying the solution with 991-ES.

Exercise 1
Calculate each of the following then check them using the 991-ES.
(i) \(1110110_2 + 1001_2 - 1101010_2\)  
(ii) \((27_8 - 25_8) - 56_8\)

Exercise 2
Calculate the following operations involving multiplication and division, if possible.
(i) \((11001_2 + 101001_2) \div 1011_2\)  
(ii) \((7A_{19} - 512_8) \times 34_8\)  
(iii) \(91A3_{16} \times 77B_{16}\)  
(iv) \(10918 + 617_8 - 3A3C_{16}\)

SOLUTIONS to Exercises
Exercise 1
(i) \(10101_2\)  
(ii) \(167_8\)

Exercise 2
(i) \(110_2\)  
(ii) \(4416E5_{16}\)  
(iii) \(731612\)
(iv) The result can only be displayed when the display mode is Dec. Otherwise the Hex, Bin or Oct mode cannot display this negative value. The answer is \(-3591\).